PROPERTIES OF sgp-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

Sgp-continuous functions are presented in this paper. Also α-continuous, strongly continuous, perfectly continuous, αg^*s-continuous, g^*-continuous functions in topological spaces are introduced. The sgp –continuous functions implies all the other continuous functions. But some of the continuous functions have not converse part. Some properties and some their examples are discussed in this paper.

KEYWORDS:

sgp- continuous, ω-continuous, α-continuous, strongly continuous, perfectly continuous, αg^*s-continuous,g^*-continuous function.

1.INDRODUCTION:

In 1963,Levine[1] introduced and studied the week forms of continuity namely semi-continuity. Balachandran [2] ,sundaram [3],crossley and Hildebrand[4], Noiri[5] have introduced g-continuity ,α-continuity,ω-continuity, gs-continuity, αg-continuity,gp-contiunity,g^*-continuity,αg^*s-continuity respectively, which are weaker forms continuous functions.

Baker[6] has obtained further properties of sgp-continuous functions. In 1982, malgnan[7] introduced and studied the concept of generalized closed functions and generalized open function, semi closed functions, semi open functions, α -open functions, ω-open and ω-closed functions, g^*-closed functions, αg^*s continuous functions.

2.PRELIMINARIES :

In the present of this paper, spaces means topological spaces on which no separation axiom is assumed explicitly stated. A be a subset of a topological space (X,τ) and (Y,σ) be a closed set and open set respectively.

2.1.DEFINITION:

A map from f: X → Y

(1) g-continuous: if f^{-1} (V) is g-open in X, for each open set V of Y.

(2) Strongly continuous: if f^{-1} (V) is both open and closed in X, for each subset V in Y.

(3) Perfectly continuous: if f^{-1} (V) is both open and closed in X, for each open set V in Y.

(4) generalized continuous: if f^{-1} (V) is g-open in X, for each open set V of Y.

(5) strongly g-continuous: if f^{-1} (V) is g-open in X, for each open set V in Y.

(6) Perfectly g-continuous: if f^{-1} (V) is both open and closed in X for each g-open set V in Y.

(7) g-closed: if f (V) is g-closed in Y, for every closed set V in X.

2.2.DEFINITION:

A function $f:(X,\tau) \to (Y,\sigma)$ is said to be **αg-continuous**, if for each open set $v \in \sigma$, f^{-1} (V) is αg-open in (X,τ).

3.sgp- Continuous Functions in Topological Spaces

sgp- Continuous Functions in Topological Spaces In this section, we introduce the concept of a new class of continuous functions are called sgp-continuous functions investigate some of their properties.

3.1.DEFINITION :

A function f: X → Y is said to be **semi generalized pre-continuous** (briefly sgp -continuous) if the inverse image of every closed set in Y is sgp-closed in X.s

3.2.THEOREM :

A function f: X → Y is sgp-continuous if and only if the inverse image of every open set in Y is sgp-open in X.

Proof:

Let F be an open set in Y. Then X - F is closed in Y. Since f is sgp-continuous, f^{-1}(X - F) is sgp-closed in X. But f^{-1} (X - F) = X- f^{-1}(F) which is sgp-closed set in X. Therefore f^{-1}(F) is sgp -open set in X.

Conversely, assume that the inverse image of every open set in Y is sgp-open in X. Let V be a closed set in Y. Then X - V is open in Y.

By hypothesis, f^{-1} (X – V) = X - f^{-1} (V) is sgp-closed set in X. So f^{-1} (V) is sgp-closed in X. Thus f is sgp-continuous function.

3.3.THEOREM :

If a function f: X →Y is continuous, then f is sgp-continuous but not conversely.

Proof:

Let V be an open set in Y. Since f is sgp- continuous function, f^{-1} (V) is open in X. And therefore f^{-1} (V) is sgp-open in X. Hence f is sgp-continuous.

3.4.EXAMPLE :

Let X = Y = {a, b, c}, τ = {X, ,{a}} and σ = {Y, ,{a},{a , c}}. Define a function f: X → Y by f (a) = c, f (b) = b and f(c) = a. Then f is sgp -continuous but not a continuous as the inverse image of closed set {b} in Y, f^{-1} ({b}) = {b} is not closed in X but it is sgp-closed in X.

3.5.DEFINITION:

A map f: X → Y is called **ω-continuous** if f^{-1} (V) is ω-open in X, for each open set V of Y

3.6.THEOREM :

Every ω-continuous function is sgp -continuous function but not conversely.

Proof:

Let g: X → Y be a ω-continuous function. Let V be a closed set in Y.

Then g^{-1}(v) is ω -closed in X as g is sgp-continuous. since g^{-1}(v) is sgp-closed in X. Hence g is sgp-continuous function.

3.7.EXAMPLE :

In f is sgp -continuous but not a ω - continuous as the inverse image of the closed set {b} in Y, f^{-1} ({b}) = {b} is not a ω -closed but it is sgp-closed in X.

3.8.DEFINITION :

A map f: X → Y **is** called **α-continuous** if is α-open in X, for each open set V in Y.

3.9.THEOREM:

Every α-continuous function is sgp -continuous function but not conversely.

Proof:

Let f: X → Y be an α -continuous function. Let F be a closed set inY.

Then f^{-1}(V) is a-closed in X. And therefore f^{-1} (V) is sgp-closed in X. Hence f is sgp-continuous function.

3.10.THEOREM:

Every αgs-continuous function is sgp-continuous but not conversely.

Proof:

Let g: X → Y be a αgs-continuous function. Let F be a closed set in Y. Then g^{-1}(V) is αgs-closed in X. Therefore g^{-1} (V) is sgp-closed in X. Therefore f is sgp-continuous function.

3.11.EXAMPLE:

Let X = Y = {a, b, c} and τ = {X,,{a, b}} and σ = {Y,,{a}, {b, c}}. Then the identity function f: X → Y is sgp -continuous but not αgs-continuous as the inverse image of the closed set {a} in Y, f^{-1} ({a}) = {a} is not a αgs-closed in X but it is sgp-closed in X.

RESULT:

The concepts of sgp-continuous functions and semi-continuous functions are independent each other as seen from the following examples.

3.12.EXAMPLE :

Let X = Y = {a, b, c} and τ = {X, , {a}, {b}, {a, b}} and σ = {Y, , {a}, {a,c}}. Define a function f: X → Y by f (a) = a, f (b) = a and f (c) = c. Then f is semi-continuous but not sgp-continuous as the inverse image of the closed set {b} in Y, f^{-1} ({b}) = {a} is not sgp-closed set in X but it is αg*s-closed in X.

3.13.EXAMPLE:

Let X = Y = {a, b, c} and τ = {X, ,{a}, {b, c}} and σ = {Y, , {a}, {b}, {a, b}}. Define a function f: X → Y by f(a) = b, f(b) = a and f(c) = c. Then f is sgp-continuous but not semi-continuous as the inverse image of the closed set {b,c} in Y, f^{-1} ({b, c}) = {a, c} is not semi-closed in X.

3.14.REMARK:

The concepts of sgp-continuous functions and g*- continuous functions are independent each other as seen from the following examples.

3.15.EXAMPLE:

Let X = Y = Z = {a, b, c} and τ = {X, , {c}, {a, c}} and σ = {Y, , {a}}. Then the identity function f: X → Y is g*-continuous function but not sgp-continuous as the inverse image of closed set {b, c} in Y, f^{-1} ({b, c}) = {b, c} is not sgp-closed in X but it is g*-closed in X.

3.16.EXAMPLE:

The function f is sgp-continuous but not a g*continuous as the inverse image of closed set {b, c} in Y, f^{-1} ({b, c}) = {a, c} is not g*-closed in X but it is sgp-closed in X. -closed in X.

3.17.THEOREM:

Every strongly continuous function is sgp -continuous.

Proof:

Let g: X → Y be a strongly continuous. Let g be a closed set in Y. Then g^{-1}(v) is both open and closed in X. Therefore g^{-1}(v) is sgp -closed in X. Hence g is sgp -continuous function.

The converse part of the theorem need not be true.

3.18.EXAMPLE:

Let X = {a, b, c} and τ = {X, , {c}, {a, c}}. σ = {Y, , {a}, {b}, {a, b}}. Define a function f: (X , τ) → (Y, σ) by f (a) = a, f (b) = c and f(c) = b. Then f is sgp -continuous but not strongly continuous, as the inverse image of open set {b} in Y, f^{-1} ({b}) ={c} is not both open and closed in X but it is sgp-open set in X.

3.19.THEOREM:

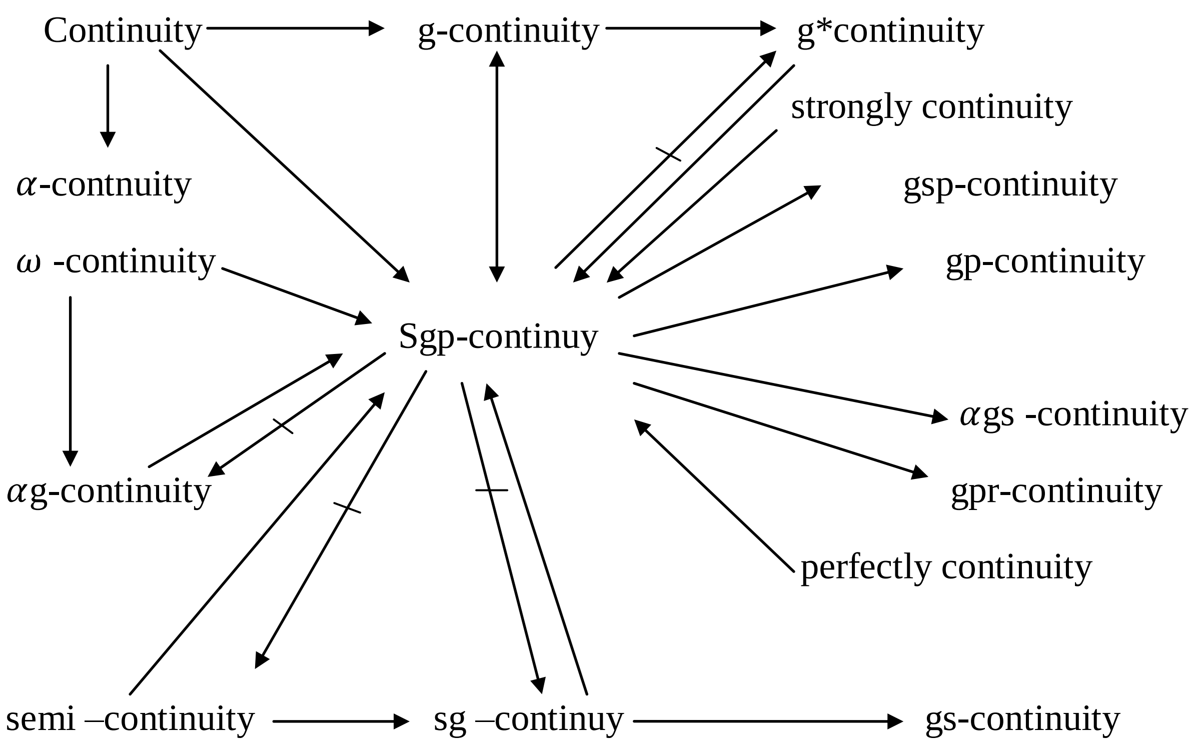
Every perfectly continuous function is sgp -continuous but not conversely.

Proof:

Let f: X → Y be a perfectly continuous function. Let G be an open set in Y. Then f^{-1} (G) is both open and closed in X. So f^{-1} (G) is open in X. Therefore f^{-1} (G) is sgp -open in X. Hence f is sgp -continuous.

3.20.REMARK:

we have the following diagram



CONCLUSION

In the present work, we have continued to study the properties of sgp-continuous function in topological space. We introduced α-continuous, strongly continuous, perfectly continuous, sgp-continuous, αg^*s-continuous, g^*-continuous function.

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