Decision Making in Sports via Possibility Fuzzy Soft Set

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Abstract. The concept of soft set is the recent topic developed for dealing with the uncertainties present in most of our real life situations. The decision making problems with imprecise data has a special significance in real life problems. In this work, an attempt has been made to apply the concept of possibility fuzzy soft set initiated by Shawkat Alkhazaleh et al, in hockey player selection process. The method involves construction of a comparison table from possibility fuzzy soft set in parametric sense for decision making.

Keywords: Generalized fuzzy soft set, Possibility fuzzy soft set, Comparison table, Membership score, Non membership score.

I Introduction

There are many complicated problems in economics, engineering, environment, social science, medical science, etc., that involve data which are not always all crisp. We cannot successfully use classical methods because of various types of uncertainties present in these problems. Uncertainty may arise due to partial information about the problem, or due to information which is not fully reliable, or due to receipt of information from more than one source. Fuzzy set theory[13], Rough set theory[11], Vague set theory[4], Intuitionistic fuzzy set theory[2] and theory of Interval valued sets are some mathematical tools to handle uncertainty arising due to vagueness. In 1999, Molodtsov [10] initiated the novel concept of soft set theory as a competitor of these theories. Soft set theory has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory.

At present, work on the soft set theory is progressing rapidly. Maji et al. [6,8] have further studied the theory of soft sets and used this theory to solve some decision-making problems. They have also introduced the concept of fuzzy soft set, a more general concept, which is a combination of fuzzy set and soft set and studied its properties[7], and also Roy and Maji used this theory to solve some decision-making problems[12]. Majumdar andSamanta[9] defined and studied the generalised fuzzy soft sets where the degree is attached with the parametrization of fuzzy sets while defining a fuzzy soft set. M.J.Borah et al.[3] used the generalized fuzzy soft in Teaching Evaluation. S.Alkhazaleh et al[1] generalized the concept of fuzzy soft sets to the possibility fuzzy soft set, in which a possibility of each element in the universe is attached with the parametrization of fuzzy sets while defining a fuzzy soft set. In this paper , we present a decision making with the help of possibility fuzzy soft set initiated by S.Alkhazaleh et al.[1],to select a best player for hockey team.

II Preliminaries

In this section we recall some definitions regarding fuzzy soft sets and possibility fuzzy soft sets which are required for this paper. Throughout our discussion, $U = \{x_1, x_2, \dots x_n\}$ would refer to an initial universal set, $E = \{e_1, e_2, \dots e_n\}$ the set of all parameters for U and I^U, the set of all fuzzy subsets of U. Also by (U, E) we mean the universal set U and the parameter set E. Let $A, B \subset E$.

Definition 1[10] A pair (F, E) is called a **soft set** on an initial universe X if and only if F is a mapping of E into the set of all subsets of the set X. In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E, F(e)$, may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set.

Definition 2[7] A pair (F, A) is called a **fuzzy soft set** over U where F is mapping given by $F : A \rightarrow I^U$, the set of all fuzzy subsets of U.

Definition 3[9] Let $F : A \rightarrow I^X$ and μ be the fuzzy subset of A, that is, $\mu : A \rightarrow I = [0, 1]$, where I^X is the collection of all fuzzy subsets of X. Let $F_\mu : A \rightarrow I^X \times I^X$ be a function defined as

$F_\mu(e) = (F(e)(x), \mu(e)(x)), \forall\, x \in X.$

Then F_μ is called possibility fuzzy soft set over the soft universe (X, E).

Here for each parameter e_i, $F_\mu(e_i) = (F(e_i)(x), \mu(e_i)(x))$ indicates not only the degree of belongingness of the elements of X in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

$F_\mu(e_i)$ may be written as follows:

$$F_\mu(e_i) = \left\{ \begin{matrix} \left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1)\right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2)\right), \\ \dots, \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n)\right) \end{matrix} \right\}$$

Example 1 Let $X = \{x_1, x_2, x_3\}$ be the set of three colleges under consideration and let E represents the level of placements got by the students in the college. Take $A \subset E$ as $A = \{e_1\ (high),\ e_2\ (normal),\ e_3\ (low)\}$. Let $\mu : A \to I^X$. Define a function $F_\mu : A \to I^X \times I^X$ as

$$F_\mu(e_1) = \left\{\left(\frac{x_1}{0.4}, 0.6\right), \left(\frac{x_2}{0.3}, 0.5\right), \left(\frac{x_3}{0.6}, 0.9\right)\right\},$$
$$F_\mu(e_2) = \left\{\left(\frac{x_1}{0.6}, 0.8\right), \left(\frac{x_2}{0.4}, 0.7\right), \left(\frac{x_3}{0.1}, 0.6\right)\right\},$$
$$F_\mu(e_3) = \left\{\left(\frac{x_1}{0.2}, 0.4\right), \left(\frac{x_2}{0.6}, 0.7\right), \left(\frac{x_3}{0.5}, 0.6\right)\right\}.$$

Then F_μ is a possibility fuzzy soft set over (X, E). In Table 1 this can be expressed as

Table 1. **F_μ in tabular form**

$F_\mu =$

	e_1	e_2	e_3
x_1	0.4,0.6	0.6,0.8	0.2,0.4
x_2	0.3,0.5	0.4,0.7	0.6,0.7
x_3	0.6,0.9	0.1,0.6	0.5,0.6

Definition 4[1] Let F_μ and G_δ be two possibility fuzzy soft set over (U,E). Then G_δ is said to be possibility fuzzy soft subset of F_μ, written as $G_\delta \cong F_\mu$, if

(i) $\delta(e)$ is fuzzy subset of $\mu(e), \forall\, e \in E$.

(ii) $G(e)$ is also a fuzzy subset of $F(e), \forall\, e \in E$.

Example 2 For the above example, define a function $G_\delta : A \to I^X \times I^X$ as

$$G_\delta(e_1) = \left\{\left(\frac{x_1}{0.3}, 0.5\right), \left(\frac{x_2}{0.1}, 0.5\right), \left(\frac{x_3}{0.4}, 0.6\right)\right\},$$
$$G_\delta(e_2) = \left\{\left(\frac{x_1}{0.4}, 0.6\right), \left(\frac{x_2}{0.2}, 0.4\right), \left(\frac{x_3}{0.1}, 0.4\right)\right\},$$
$$G_\delta(e_3) = \left\{\left(\frac{x_1}{0.0}, 0.3\right), \left(\frac{x_2}{0.5}, 0.2\right), \left(\frac{x_3}{0.3}, 0.4\right)\right\}.$$

Then G_δ possibility fuzzy soft subset of F_μ.

Definition 5[1] Let F_μ be the possibility fuzzy soft set over (U, E). Then $1 - F_\mu$ is said to be the complement of F_μ and is defined as $1 - F_\mu = G_\delta$ such that $\delta(e) = (1 - \mu)(e)$, and

$G(e) = (1 - F)(e), \forall\, e \in E.$

Example 3 For the example 1.4.3.2, $1 - F_\mu = G_\delta$ is given by

$$G_\delta(e_1) = \left\{\left(\frac{x_1}{0.6}, 0.4\right), \left(\frac{x_2}{0.7}, 0.5\right), \left(\frac{x_3}{0.4}, 0.1\right)\right\},$$
$$G_\delta(e_2) = \left\{\left(\frac{x_1}{0.4}, 0.2\right), \left(\frac{x_2}{0.6}, 0.3\right), \left(\frac{x_3}{0.9}, 0.4\right)\right\},$$
$$G_\delta(e_3) = \left\{\left(\frac{x_1}{0.8}, 0.6\right), \left(\frac{x_2}{0.4}, 0.3\right), \left(\frac{x_3}{0.5}, 0.4\right)\right\}.$$

In Table 2 this can be expressed as

Table 2 $(1-F_\mu)$ **in tabular form**

$1-F_\mu =$

	e_1	e_2	e_3
x_1	0.6,0.4	0.4,0.2	0.8,0.6
x_2	0.7,0.5	0.6,0.3	0.4,0.3
x_3	0.4,0.1	0.9,0.4	0.5,0.4

Definition 6[1] Let F_μ and G_δ be two possibility fuzzy soft set over (U, E). Then F_μ and G_δ is said to be equal, written as $F_\mu = G_\delta$, if $F_\mu \widetilde{\subseteq} G_\delta$, and $G_\delta \widetilde{\subseteq} F_\mu$. In other words, $F_\mu = G_\delta$, if the following conditions are satisfied

(i) $\mu(e)$ is equal to $\delta(e)$, $\forall\, e \in E$.

(ii) $F(e)$ is equal to $G(e)$, $\forall\, e \in E$.

III Decisions Making In Sports Via Possibility Fuzzy Soft Set

Alkhazaleh et al (2011) generalized the concept of fuzzy soft sets to the possibility fuzzy soft set, in which a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. This section presents a decision making with the help of possibility fuzzy soft set to select the best player for hockey team. A hockey team is looking for a player for a specific position that they need to cover. Obviously, there are lots of choices but not all of them can be covered as a real choice. Let U be the universal set of players and the parameter set E represents the traits needed for a hockey player.

A. Algorithm

The following algorithm may be used to select the best player for the hockey team.

Step 1 : Input the possibility fuzzy soft set F_μ

Step 2 : Consider F_μ in tabular form

Step 3 : Compute the complement $1-F_\mu$

Step 4 : Consider $1-F_\mu$ in tabular form

Step 5 : Compute the comparison table for F_μ and $1-F_\mu$

Step 6 : Compute the membership score

Step 7 : Compute nonmember ship score

Step 8 : Compute the final score

Step 9 : Find the lowest score.

Comparison table is obtained by multiplying each entry of the table representing the possibility fuzzy soft set by the corresponding value of $\mu(e_i)$. Finally find the lowest value from the final score table, which would correspond to the best player.

B. Numerical Example

Suppose that there are players with five traits namely, stick handling, skating, shooting, passing and checking.

Let $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ be the set of six players and $A = \{e_1$(stick handling), e_2(skating), e_3(shooting), e_4(passing), e_5(checking)$\}$ in E. Let F_μ be the possibility fuzzy soft set represents the level of each traits of each of the six players.

Consider the possibility fuzzy soft set F_μ as follows:

$$F_\mu(e_1) = \left\{ \begin{matrix} \left(\frac{P_1}{0.2}, 0.8\right), \left(\frac{P_2}{0.4}, 0.3\right), \left(\frac{P_3}{0.4}, 0.5\right), \\ \left(\frac{P_4}{0.3}, 0.6\right), \left(\frac{P_5}{0.6}, 0.5\right), \left(\frac{P_6}{0.1}, 0.2\right) \end{matrix} \right\},$$

$$F_{\mu}(e_2) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.4}, 0.7\right), \left(\frac{P_2}{0.6}, 0.4\right), \left(\frac{P_3}{0.7}, 0.2\right), \\ \left(\frac{P_4}{0.8}, 0.3\right), \left(\frac{P_5}{0.4}, 0.3\right), \left(\frac{P_6}{0.4}, 0.1\right) \end{array} \right\},$$

$$F_{\mu}(e_3) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.2}, 0.3\right), \left(\frac{P_2}{0.3}, 0.7\right), \left(\frac{P_3}{0.1}, 0.5\right), \\ \left(\frac{P_4}{0.4}, 0.2\right), \left(\frac{P_5}{0.1}, 0.4\right), \left(\frac{P_6}{0.8}, 0.3\right) \end{array} \right\},$$

$$F_{\mu}(e_4) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.7}, 0.2\right), \left(\frac{P_2}{0.1}, 0.4\right), \left(\frac{P_3}{0.4}, 0.7\right), \\ \left(\frac{P_4}{0.1}, 0.7\right), \left(\frac{P_5}{0.2}, 0.6\right), \left(\frac{P_6}{0.3}, 0.9\right) \end{array} \right\},$$

$$F_{\mu}(e_5) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.8}, 0.3\right), \left(\frac{P_2}{0.2}, 0.6\right), \left(\frac{P_3}{0.3}, 0.5\right), \\ \left(\frac{P_4}{0.5}, 0.4\right), \left(\frac{P_5}{0.5}, 0.1\right), \left(\frac{P_6}{0.3}, 0.4\right) \end{array} \right\},$$

F_{μ} can be expressed as tabular form in Table 3.

Table 3 F_{μ} in tabular form

	e_1	e_2	e_3	e_4	e_5
P_1	0.2,0.8	0.4,0.7	0.2,0.3	0.7,0.2	0.8,0.3
P_2	0.4,0.3	0.6,0.4	0.3,0.7	0.1,0.4	0.2,0.6
P_3	0.4,0.5	0.7,0.2	0.1,0.5	0.4,0.7	0.3,0.5
P_4	0.3,0.6	0.8,0.3	0.4,0.2	0.1,0.7	0.5,0.4
P_5	0.6,0.5	0.4,0.3	0.1,0.4	0.2,0.6	0.5,0.1
P_6	0.1,0.2	0.4,0.1	0.8,0.3	0.3,0.9	0.3,0.4

Consider the complement possibility fuzzy soft set $(1 - F_{\mu})$ as follows:

$$(1 - F_{\mu})(e_1) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.8}, 0.2\right), \left(\frac{P_2}{0.6}, 0.7\right), \left(\frac{P_3}{0.6}, 0.5\right), \\ \left(\frac{P_4}{0.7}, 0.4\right), \left(\frac{P_5}{0.4}, 0.5\right), \left(\frac{P_6}{0.9}, 0.8\right) \end{array} \right\},$$

$$(1 - F_{\mu})(e_2) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.6}, 0.3\right), \left(\frac{P_2}{0.4}, 0.6\right), \left(\frac{P_3}{0.3}, 0.8\right), \\ \left(\frac{P_4}{0.2}, 0.7\right), \left(\frac{P_5}{0.6}, 0.7\right), \left(\frac{P_6}{0.6}, 0.9\right) \end{array} \right\},$$

$$(1 - F_{\mu})(e_3) = \left\{ \begin{array}{l} \left(\frac{P_1}{0.8}, 0.7\right), \left(\frac{P_2}{0.7}, 0.3\right), \left(\frac{P_3}{0.9}, 0.5\right), \\ \left(\frac{P_4}{0.6}, 0.8\right), \left(\frac{P_5}{0.9}, 0.6\right), \left(\frac{P_6}{0.2}, 0.7\right) \end{array} \right\},$$

$$(1-F_\mu)(e_4)=\left\{\begin{array}{c}\left(\frac{P_1}{0.3},0.8\right),\left(\frac{P_2}{0.9},0.6\right),\left(\frac{P_3}{0.6},0.3\right),\\ \left(\frac{P_4}{0.9},0.3\right),\left(\frac{P_5}{0.8},0.4\right),\left(\frac{P_6}{0.7},0.1\right)\end{array}\right\},$$

$$(1-F_\mu)(e_5)=\left\{\begin{array}{c}\left(\frac{P_1}{0.2},0.7\right),\left(\frac{P_2}{0.8},0.4\right),\left(\frac{P_3}{0.7},0.5\right),\\ \left(\frac{P_4}{0.5},0.6\right),\left(\frac{P_5}{0.5},0.9\right),\left(\frac{P_6}{0.7},0.6\right)\end{array}\right\},$$

$(1-F_\mu)$ can be expressed as tabular form in Table 4.

Table 4 $(1-F_\mu)$ in tabular form

	e_1	e_2	e_3	e_4	e_5
P_1	0.8,0.2	0.6,0.3	0.8,0.7	0.3,0.8	0.2,0.7
P_2	0.6,0.7	0.4,0.6	0.7,0.3	0.9,0.6	0.8,0.4
P_3	0.6,0.5	0.3,0.8	0.9,0.5	0.6,0.3	0.7,0.5
P_4	0.7,0.4	0.2,0.7	0.6,0.8	0.9,0.3	0.5,0.6
P_5	0.4,0.5	0.6,0.7	0.9,0.6	0.8,0.4	0.5,0.9
P_6	0.9,0.8	0.6,0.9	0.2,0.7	0.7,0.1	0.7,0.6

The following comparison tables are obtained by multiplying each entry of the table representing the possibility fuzzy soft set by the corresponding value of $\mu(e_i)$.

Comparison table for possibility fuzzy soft set F_μ is given in Table 5.

Table 5. F_μ - Comparison table

	e_1	e_2	e_3	e_4	e_5
P_1	0.16	0.28	0.06	0.14	0.24
P_2	0.12	0.24	0.21	0.04	0.12
P_3	0.20	0.14	0.05	0.28	0.15
P_4	0.18	0.24	0.08	0.07	0.20
P_5	0.30	0.12	0.04	0.12	0.05
P_6	0.02	0.04	0.24	0.27	0.12

Comparison table for complement possibility fuzzy soft set $1-F_\mu$ is given in Table 6.

Table 6 $(1-F_\mu)$ - Comparison table

	e_1	e_2	e_3	e_4	e_5
P_1	0.16	0.18	0.56	0.24	0.14
P_2	0.42	0.24	0.21	0.54	0.32
P_3	0.30	0.24	0.45	0.18	0.35
P_4	0.28	0.14	0.48	0.27	0.30

P_5	0.20	0.42	0.54	0.32	0.45
P_6	0.72	0.54	0.14	0.07	0.42

The membership and non-membership score values are given in Tables 7 and 8.

Table 7 Membership score table

	Row Sum
P_1	0.88
P_2	0.73
P_3	0.82
P_4	0.77
P_5	0.63
P_6	0.69

Table 8 Non - membership score table

	Row Sum
P_1	1.28
P_2	1.73
P_3	1.52
P_4	1.47
P_5	1.93
P_6	1.89

The final score computation is given in Table 9.

Table 9 Final score table

	Membership Score (m)	**Non-Membership Score (n)**	**Final Score ($m + n - mn$)**
P_1	0.88	1.28	1.0336
P_2	0.73	1.73	1.1971
P_3	0.82	1.52	1.0936
P_4	0.77	1.47	1.1081
P_5	0.63	1.93	1.3441
P_6	0.69	1.89	1.2759

The lowest score is 1.0336 and it corresponds to the player P_1. Thus the player P_1 is the best player to be selected for the hockey team among all the players under consideration.

IV Conclusion

In this paper we have applied the notion of possibility fuzzy soft set in developing a model for a hockey player selection. It is hoped that our model would certainly end up with a decision that would be nearest to the desired objective. It is expected that the approach will be useful to handle other realistic uncertain problem.

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