FUZZY BRIDGE AND FUZZY TREE IN FUZZY GRAPH THEORY

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ABSTRACT

In this paper, we survey on fuzzy bridge and fuzzy tree in fuzzy graph theory . Throught this paper (σ,μ) represent fuzzy graph on which no separation axioms are assumed unless otherwise mentioned. Here , we introduced bridge and tree in fuzzy sets and derive these fuzzy sets by using some basic definitions and theorems. Further we can extend these results are in fuzzy cycles.

Key words :

Fuzzy bridge, Fuzzy tree, Fuzzy vertex ,cut vertex ,arc, complete fuzzy graph.

1.Introduction :

Fuzzy set theory is very much a paradigm shift that first gained acceptance in the far east and its successful application as ensured its adoption around the world.In 1965 Loft A.Zadeh introduced the notion of fuzzy subset of a set as a method for representing uncertainty.The field grew enormously finding applications in areas diverse as washing machine to handwriting recognition.It has also come to include the theory of fuzzy algebra, fuzzy graph, fuzzy working on concepts fuzzy tree, domination in fuzzy graph and so on. Based on the definitions of fuzzy sets and fuzzy relations,Azriel Rosenfeld defined fuzzy graph and studied many properties.

2. Preliminaries :

2.1 Definition

A fuzzy set of a base set V is specified by its membership function σ, where $\sigma : V \rightarrow [0,1]$ assigning to each a $\in$ V the degree or grade to which a $\in \sigma$.The membership function of a fuzzy set σ of V is called **fuzzy set**.

2.2 Definition

A **fuzzy graph** G = (σ, μ) is a pair of function σ : V → [0,1] and μ : V × V → [0,1] , where for all a,b ∈ V, we have

$$\mu(a,b) \leq \sigma(a) \wedge \sigma(b) .$$

2.3 Definition

A **strong fuzzy graph** G = (σ, μ) is a pair of function σ : V → [0,1] and μ : V × V → [0,1] , where for all a,b ∈ V, we have

$$\mu(a,b) = \sigma(a) \wedge \sigma(b) .$$

2.4 Definition

An arc (a,b) is a **fuzzy bridge** of G : (σ, μ) if the deletion of (a,b) reduces the strength of connectedness between some pair of vertex.

Equivalently , (a,b) is a fuzzy bridge iff there are vertex x,y such that (a,b) is an arc of every strongest x-y path.

2.5 Definition

A vertex is a **fuzzy cut vertex** of G : (σ, μ) if removal of its reduce the strength of connectedness between some pair of vertices.

Equivalently, c is a fuzzy cut vertex iff there exit a,b distinct from c such that c is on every strongest a-b path.

2.6 Definition

A connected fuzzy graph G : (σ, μ) is a **fuzzy tree** if it has a fuzzy spanning subgraph T:(a,b) which is a tree where all arcs (a,b) not in T.

$$\mu(a, b) < b^{\infty}(a,b).$$

Equivalently, there is a path in T between a and b whose strength exceeds $\mu(a, b)$ for all (a,b) not in T.

Note:

If G is such that G^* is a tree then T is G itself.

2.7 Definition

Let G : (σ, μ) is a fuzzy graph such that G^* is a cycle.Then G is called a **fuzzy cycle** if it has more than one weakest arc.

2.8 Definition

A complete fuzzy graph is a fuzzy graph G = (σ, μ) such that $\mu(a, b)$ = $\sigma(a) \wedge \sigma$(b) for all a,b ∈ σ^*. If G = (σ, μ) is a complete fuzzy graph then μ^∞ = μ and G has no fuzzy cut vertices.

Theorem 2.1

Let G be a connected graph and let x,y be any two vertices in V.Then there exist a strong path from x to y.

Proof :

Assume that , G be connected and let x,y be any two vertices in V.

Since G is connected, there exist a path from x to y such that,

$$\mu((x_{i-1,}\ x_i) > 0 \text{ for all } 1 \leq i \leq n.$$

If $\mu(x_{j-1,}\ x_j)$ is not strong for some 1≤ j ≤ n.

We must have $\mu((x_{j-1,}\ x_j)$ < $(\mu^\infty(x_{j-1,}\ x_j)$.

Hence there exit a path a path e_j from $(x_{j-1,}\ to\, x_j)$ where strength is greater than $\mu\ (x_{j-1,}\ x_j)$, so that all its arcs have weight greater than$\mu\ (x_{j-1,}\ x_j)$.

If some are on e_j is not strong this argument can be repeated.

Evidently , the argument cannot be repeated arbitrarily often.

Thus eventually we can find a path from x to y on which all the arcs are stong.

Hence the proof.

Theorem 2.2

Let G : (σ, μ) be a fuzzy graph and let (a,b) be a fuzzy bridge of G.

Then μ^∞(a,b) = μ(ab).

Proof :

Suppose that, (a,b) is a fuzzy bridge and that μ^∞(a,b) exceed μ(ab).

Then there exist a strongest a-b path with strength greater than μ(ab) and all arcs of this strongest path have strength greater than μ(ab).

Now this path together with the arc (ab) forms a cycle in which (ab) is the weakest arc.

Contradicting that (ab) is a fuzzy bridge Graph.

Hence the proof.

Example:

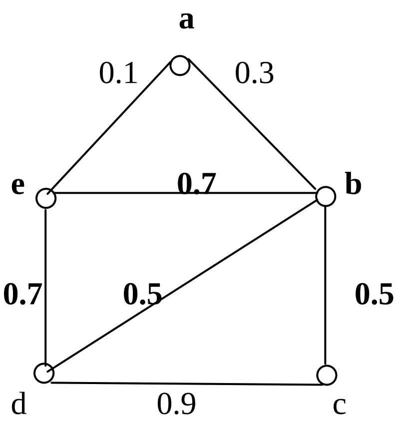

fig 2.1 (a,b), (b,e),(e,d), and (d,c) are fuzzy bridges.

Theorem 2.3

Let G : (σ, μ) be a complete fuzzy graph then for any edge ab $\in \mu^*$, μ^∞(a,b) = μ(ab).

Proof :

By definition,

$$\mu^2(a,b) = \vee_{z \in \sigma^*}\{\mu(az) \wedge \mu(zb)\}$$
$$= \vee\{\sigma(a) \wedge \sigma(b) \wedge \sigma(z)\}$$
$$= \sigma(a) \wedge \sigma(b).$$
$$= \mu(ab)$$

Similarly

μ^3(a,b) = μ(ab) in the same way one can show that

μ^k(a,b) = μ(ab) for all positive integers k.

Thus μ^∞(a,b) = sup{ μ^k(a,b) for all integers k $\geq$ 1}= μ(ab).

Theorem 2.4

Let G : (σ, μ) be a fuzzy graph and let c be common vertex of atleast two fuzzy bridges, then c is a fuzzy cut vertex.

Proof :

Let (a_1,c) and (c,a_2) be two fuzzy bridges. Then there exist some a,b such that (a,c) is on every strongest a-b path.

If c is distinct from a and b it follows that c is a fuzzy cut vertex.

Next suppose one of b, a is c so that (a,c) is on every strongest a-c path.

If possible let c be not a fuzzy cut vertex.

Then between every two vertices distinct from c, there exist atleast one strongest path P not containing c.

In particular there exists atleast one strongest path P joining a_1 and a_2 not containing c

This path together with (a_1, c) and (c,a_2) forms a cycle.

Now we have the following two cases:

Case: 1

a_1, c, a_2 is not a strongest path.

Then clearly either (a,c) or (c,a_2) or both become the weakest arcs of the cycle which contradicts that (a_1,c) and (c,a_2) are fuzzy bridges.

Case: 2

a_1, c, a_2 is a strongest path joining a_1 to a_2 .

Then μ^{∞}(a,b) = $\mu(a_1,c) \wedge \mu(c,a_2)$. The strength of P.

The arc of P are atleast as strong as $\mu(a_1,c)$ and $\mu(c,a_2)$.

Which implies that, $\mu(a_1,c)$, $\mu(c,a_2)$ or both are the weakest arcs of the cycle.

Which is again contradiction.

Hence the proof.

Theorem 2.5

If G is a fuzzy tree. Then interval vertices of F are the fuzzy cut vertices of G.

Proof :

Let c be any vertex in G but which is not an end vertex of T.

Since it is the common vertex of atleast two arcs in T, which are fuzzy bridges of G.

Hence c is a fuzzy cut vertex.

Also if c is an end vertex of T then c is not a fuzzy cut vertex for if so there exist a,b distinct from c such that c is on every strongest a-b path and one such path certainly lies in T.

But c being an end node of T.

This is not possible. Hence the internal vertex of T are the fuzzy cut vertices of G.

Example :

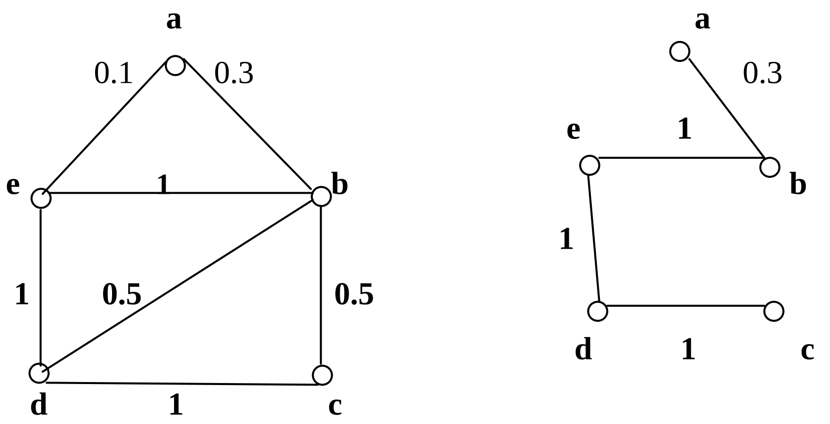

fig 2.1 fuzzy tree

Theorem 2.6

Let G : (σ, μ) be a connected fuzzy graph with no fuzzy cycle.Then G is a fuzzy tree .

Proof :

If G* has no fuzzy cycles then G^* is a tree and G is a fuzzy tree. So assume that G has cycles and by hypothesis no cycles is a fuzzy cycles.

i.e) every cycle in G will have exactly one weakest arc in it.

Remove the weakest arc(say) e in a cycle C of G. If there are still cycles in the resulting fuzzy graph repeat the process, which eventually results in a fuzzy sub graph which is a tree and which is the required spanning subgraph T.

Hence the proof.

3. CONCLUSION

In this paper , we summarise the fuzzy bridges and fuzzy trees in fuzzy graph theory.We derived the characterization of fuzzy bridge by using some theorems and example.

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